March 26, 1997 WOCOMAL VARSITY MEET

ROUND I: Elementary number theory

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. What is the smallest prime number p such that 2^p+3 is not prime?

2. In a numeration system with a positive integral base, the numbers 104_b and 241_b represent the degree measures of a pair of supplementary angles. Find the base of this numeration system (Supplementary angles add up to 180°).

3. How many positive integers divide both 5852 and 7315?

ANSWERS

(1 pt) 1._____

(2 pts) 2._____

(3 pts) 3._____

Hudson, Tahanto

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ROUND II: Algebra I - open

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. The sum of two numbers is 8. Their difference is 12. Find the smaller one.

2. If $3z + 7t = 22$ and z and t are positive integers, find all possible values of z.

3. Solve $\sqrt{2x+1} - \sqrt{x-3} = 2$ for x

ANSWERS
(1 pt) 1._____

(2 pts) 2._____

(3 pts) 3._____

Southbridge, Tantasqua, Westboro

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ROUND III: Theory of polynomial equations and functions, complex numbers

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Solve: $(2x-7)(5x-3)=5x-3$

2. The polynomial x^3+6x^2+cx+d is exactly divisible by both $x-1$ and $x-2$. What must be the value of $c+d$?

3. If $3+i$ is a root of $x^3-8x^2+22x+k=0$, what is the value of k?

ANSWERS

(1 pt) 1._____

(2 pts) 2._____

(3 pts) 3._____

Leicester, Tantasqua, Westboro

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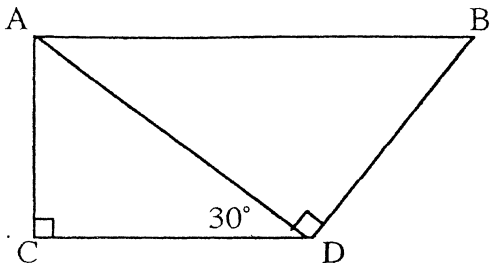
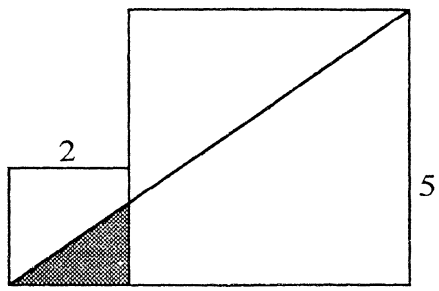
ROUND IV : Similarity and Pythagorean relationships

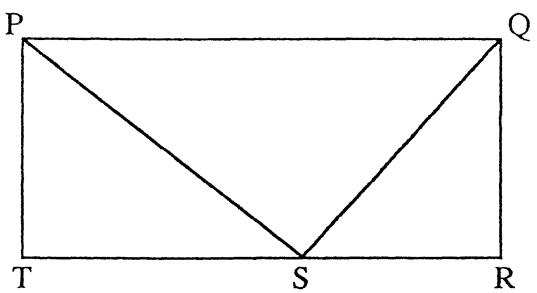
ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. 

$\overline{AB} \parallel \overline{CD}$ and AB = 12.
Find CD.

2. There are two squares with dimensions shown. Find the area of the shaded triangle, as a fraction, mixed number, or simplified radical, not as a decimal.



3. 

PS = 40, QS = 30, PQ = 50 and PQRT is a rectangle. Find length RS.

ANSWERS
(1 pt) 1._____

(2 pts) 2._____

(3 pts) 3._____

Burncoat, Clinton, Hudson

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ROUND V: Trigonometry - open

ALL ANSWERS MUST BE IN THE FORM SPECIFIED IN THE PROBLEM

1. Find, in degrees, the least positive integer x for which $\left(2^{\sin^2 x}\right)\left(2^{\cos^2 x}\right)\left(2^{\tan^2 x}\right) = 2^2$

2. A flat triangular lot faces two streets that meet at an 85° angle. The sides of the lot along the streets are each 160 feet long. Find the perimeter of the lot to the nearest foot.

3. Segment $\overline{QT}$ is tangent at point Q to a circle with center P. Angle TPQ intercepts arc s on circle P. If PQ = 5 and QT = 12, find the length of arc s, to the nearest 0.01.

ANSWERS

(1 pt) 1._____

(2 pts) 2._____ ft

(3 pts) 3._____

Doherty, Tahanto, Worcester Academy

March 26, 1997 WOCOMAL VARSITY MEET

TEAM ROUND: Topics of previous rounds and open.

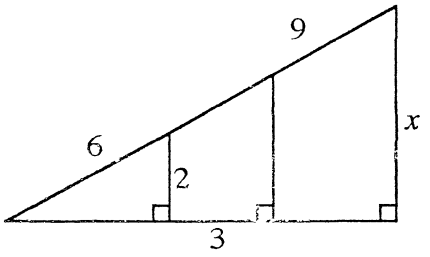
ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM EXCEPT THAT NUMBER 4 MAY BE ANSWERED WITH A DECIMAL FORM ROUNDED TO THE NEAREST .001

2 points each

1. Find the least prime number greater than 800.

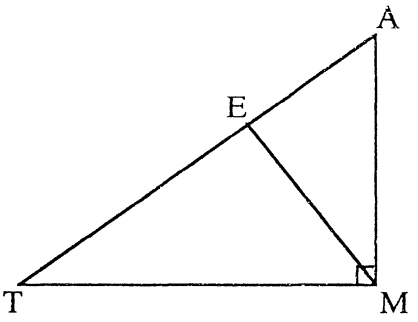
2. If $f(x) = \dfrac{x+1}{x}$, find a simplified rule for $f(f(f(x)))$

3. A function f has the property that $f(xy) = f(x) + f(y)$. If $f(2) = a$ and $f(3) = b$, what (in terms of a and b) is $f(108)$?

4. 

Not to scale. Find x.

5. In right triangle TAM, TA = 25, EM = 12, ET>EA, and $\overline{TA} \perp \overline{EM}$. Find the value of sin T.



6. What is the probability that an arrangement of the letters of the word "ides" will be either "dies" or "side"? Give your answer as a reduced fraction.

7. The bisectors of the exterior angles formed at the vertices of the acute angles of a right triangle meet at point A. What is the measure of the acute angle formed at A?

8. Point P(99,100) is reflected across the line $y = -x$ to get point Q. State both coordinates of Q.

9. Consider an extended "Pythagorean" equation $a^2 + b^2 + c^2 = d^2$. If a, b, c, and d are all counting numbers and different from one another, what is the smallest possible value for d ?

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)UND I	1. 1 pt	5	
# thry	2. 2 pts	7	
	3. 3 pts	8	
ROUND II	1. 1 pt	-2	
alz l	2. 2 pts	5	
	3 3 pts	4, 12	need both
ROUND III	1. 1 pt	4, $\frac{3}{5}$	need both
ooly ury	2. 2 pts	-7	
	3. 3 pts	-20	
ROUND IV	1. 1 pt	9	
sim Pyth	2. 2 pts	$1\frac{3}{7}$ or $\frac{10}{7}$	not decimal form
	3. 3 pts	18	
ROUND V	1. 1 pt	45°	may omit $^\circ$
trig	2. 2 pts	536	need this integer
	3. 3 pts	5.88	need 2 decimal places

TEAM ROUND 2 pts each

1. 809
2. $\frac{2+3x}{1+2x}$
3. $2a+3b$
4. $5+\frac{3\sqrt{2}}{4}$ OR $\frac{20+3\sqrt{2}}{4}$ OR 6.061
5. $\frac{3}{5}$ OR .6
6. $\frac{1}{12}$ no decimals
7. 45° OR $\frac{\pi}{4}$
8. $(-100, -99)$
9. 7